

### C3 DIFFERENTIATION

### Answers - Worksheet 1

- 1**
- a**  $\frac{1}{2}y^{-\frac{1}{2}}$
- b**  $y = x^2$
- c**  $2x$
- d**  $\frac{dx}{dy} = \frac{1}{2}y^{-\frac{1}{2}} = \frac{1}{2\sqrt{y}} = \frac{1}{2x}$
- $$\frac{1}{\left(\frac{dx}{dy}\right)} = \frac{1}{\left(\frac{1}{2x}\right)} = 2x \therefore \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$$
- 2**
- a**  $\frac{dy}{dx} = 2e^{2x-1}$
- $x = \frac{1}{2}(\ln y + 1)$
- $$\frac{dx}{dy} = \frac{1}{2y} = \frac{1}{2e^{2x-1}}$$
- $$\frac{dy}{dx} \times \frac{dx}{dy} = 2e^{2x-1} \times \frac{1}{2e^{2x-1}} = 1$$
- b**  $\frac{dy}{dx} = 3x^2$
- $x = (y-2)^{\frac{1}{3}}$
- $$\frac{dx}{dy} = \frac{1}{3}(y-2)^{-\frac{2}{3}} = \frac{1}{3x^2}$$
- $$\frac{dy}{dx} \times \frac{dx}{dy} = 3x^2 \times \frac{1}{3x^2} = 1$$
- c**  $\frac{dx}{dy} = \frac{1}{2}(\ln y)^{-\frac{1}{2}} \times \frac{1}{y} = \frac{1}{2y\sqrt{\ln y}}$
- $y = e^{x^2}$
- $$\frac{dy}{dx} = 2xe^{x^2} = 2y\sqrt{\ln y}$$
- $$\frac{dy}{dx} \times \frac{dx}{dy} = 2y\sqrt{\ln y} \times \frac{1}{2y\sqrt{\ln y}} = 1$$
- 3**
- a**  $\frac{dx}{dy} = 2y$
- $$\therefore \frac{dy}{dx} = \frac{1}{2y}$$
- b**  $\frac{dx}{dy} = 3(y-1)^2 \times 1$
- $$\therefore \frac{dy}{dx} = \frac{1}{3(y-1)^2}$$
- c**  $\frac{dx}{dy} = \sec^2 y$
- $$\therefore \frac{dy}{dx} = \cos^2 y$$
- d**  $\frac{dx}{dy} = \frac{1}{3y+2} \times 3$
- $$\therefore \frac{dy}{dx} = \frac{3y+2}{3}$$
- e**  $\frac{dx}{dy} = 2 \sin y \cos y = \sin 2y$
- $$\therefore \frac{dy}{dx} = \operatorname{cosec} 2y$$
- f**  $\frac{dx}{dy} = \frac{1 \times e^y - (y-2) \times e^y}{(e^y)^2} = \frac{3-y}{e^y}$
- $$\therefore \frac{dy}{dx} = \frac{e^y}{3-y}$$
- 4**
- a**  $\frac{dx}{dy} = 3y^2 - 8y$
- b**  $y = 3 \therefore x = -9$
- $$\frac{dx}{dy} = 3$$
- $$\therefore \text{grad} = \frac{dy}{dx} = \frac{1}{3}$$
- $$\therefore y - 3 = \frac{1}{3}(x + 9)$$
- $$[y = \frac{1}{3}x + 6]$$
- 5**
- a**  $e^y = ax + b$
- $$x = \frac{1}{a}(e^y - b)$$
- b**  $\frac{dx}{dy} = \frac{1}{a}e^y$
- c**  $\frac{d}{dx} [\ln(ax + b)] = \frac{dy}{dx} = 1 \div \frac{dx}{dy}$
- $$= \frac{a}{e^y} = \frac{a}{ax + b}$$
- 6**
- a**  $\ln y = \ln 3^x = x \ln 3$
- $$\therefore x = \frac{\ln y}{\ln 3}$$
- b**  $\frac{dx}{dy} = \frac{1}{\ln 3} \times \frac{1}{y} = \frac{1}{y \ln 3}$
- c**  $\frac{dy}{dx} = 1 \div \frac{dx}{dy} = y \ln 3$
- $$= 3^x \ln 3$$
- d**  $\text{grad} = 9 \ln 3$
- $$\therefore y - 9 = (9 \ln 3)(x - 2)$$
- $$[y = 9x \ln 3 + 9 - 18 \ln 3]$$